Ratio of Hadronic Decay Rates of J/ψ and $\psi(2S)$ and the $\rho\pi$ Puzzle

Y. F. Gu[(1),†], X. H. Li[(2)]
(1)Institute of High Energy Physics, Beijing 100039, People's Republic of China
(2)Physics Department, University of California, Riverside, CA 92521, USA
(February 1, 2008)

The so-called $\rho\pi$ puzzle of J/ψ and $\psi(2S)$ decays is examined using the experimental data available to date. Two different approaches were taken to estimate the ratio of J/ψ and $\psi(2S)$ hadronic decay rates. While one of the estimates could not yield the exact ratio of $\psi(2S)$ to J/ψ inclusive hadronic decay rates, the other, based on a computation of the inclusive ggg decay rate for $\psi(2S)$ (J/ψ) by subtracting other decay rates from the total decay rate, differs by two standard deviations from the naive prediction of perturbative QCD, even though its central value is nearly twice as large as what was naively expected. A comparison between this ratio, upon making corrections for specific exclusive two-body decay modes, and the corresponding experimental data confirms the puzzles in J/ψ and $\psi(2S)$ decays. We find from our analysis that the exclusively reconstructed hadronic decays of the $\psi(2S)$ account for only a small fraction of its total decays, and a ratio exceeding the above estimate should be expected to occur for a considerable number of the remaining decay channels. We also show that the recent new results from the BES experiment provide crucial tests of various theoretical models proposed to explain the puzzle.

PACS numbers: 13.25.Gv, 12.38.Qk

One of the outstanding problems in heavy quarkonium physics is the strong suppression of the $\psi(2S)$ decays to vector plus pseudoscalar-meson (VP) final states, $\rho\pi$ and $K^+\overline{K}^{*-} + c.c.$, which is referred to as the $\rho\pi$ puzzle [1]. Following the first observation of this anomaly [2], meagre experimental progress was made over the years, and theoretical analysis based on limited data often lead to unsatisfactory, sometimes premature, inferences. The situation has been changed dramatically in the last few years. A wealth of interesting new information, which extended the puzzle considerably, has emerged from intense studies of $\psi(2S)$ hadronic decays at the BES experiment, using a large sample of 3.79 million $\psi(2S)$ decays [3]. It is hoped for that new concerted efforts on both the theoretical and experimental side would eventually lead to a solution of this long-standing conundrum.

In this paper we seek to examine the $\rho\pi$ puzzle based purely on existing experimental data. We begin with an analysis for estimating the ratio of hadronic decay rates of J/ψ and $\psi(2S)$, which we shall denote by Q, by using the data compiled by the Particle Data Group [4], in an attempt to avoid as many theoretical ambiguities as possible in the analysis. Two different approaches to this estimate are performed. First we compare the results between themselves, and then the naive prediction of perturbative QCD (PQCD) is used. Subsequently, possible corrections to Q are discussed, as they associate with specific exclusive decay modes, and the corrected values of Q are used as standards to compare with the corresponding experimental data. Comments on the issue of J/ψ and $\psi(2S)$ decays to multi-hadron final states and on the potential similarity of the η_c-$\eta_c(2S)$ decays to the $\rho\pi$ puzzle are profusely added. Finally, various theoretical models are discussed in order to offer a greater understanding of the $\rho\pi$ puzzle, in light of recent BES results.

Conventionally, measured ratios of $\psi(2S)$ to J/ψ branching fractions for specific exclusive hadronic decays are compared with the naive prediction of PQCD, the so-called "15% rule". In the framework of PQCD [5], one expects $J/\psi(\psi(2S))$ to decay to hadrons via three gluons, or a single direct photon. In either case, the partial width of the decay is proportional to $|\Psi(0)|^2$, where $|\Psi(0)|$ is the wave function at the origin in the nonrelativistic quark model of the $c\bar{c}$ quark state. Thus one finds that

$$Q_h \equiv \frac{B(\psi(2S) \to ggg)}{B(J/\psi \to ggg)} = \frac{\alpha_s^3(\psi(2S))}{\alpha_s^3(J/\psi)} \frac{B(\psi(2S) \to e^+e^-)}{B(J/\psi \to e^+e^-)} = (14.8 \pm 2.2)\%, \tag{1}$$

where the new world averages of the leptonic branching fractions are used (see Table 1). This is assuming that the strong coupling constants are equivalent, i.e., $\alpha_s(\psi(2S))$=$\alpha_s(J/\psi)$. Taking the running constant α_s into account [6], the ratio Q_h becomes

$$Q_h = (12.5 \pm 1.9)\%. \tag{2}$$

The MARKII experiment first compared the theoretical prediction of this value ((12.2 ± 2.4)% then used) with measurements for a number of exclusive hadronic decays of the J/ψ and the $\psi(2S)$, thus revealing the $\rho\pi$ puzzle [2].

However, this naive prediction suffers several apparent approximations. Higher order corrections, which may not even be small, are not included in this calculation. For example, a first order correction to the branching fraction of $J/\psi \to e^+e^-$ could be 50% of the lowest term if one were to use $\alpha_s(m_{J/\psi}) \sim 0.2$ [7]. The relativistic effect is also ignored. Since the mass difference between J/ψ and $\psi(2S)$ is around 20% and $< v^2/c^2 >\sim 0.24$ for J/ψ, this correction may be at the same level as the lowest order [7]. The inclusion of the finite size of the decay vertex will significantly reduce the ggg decay width of J/ψ [8]. Moreover, the effect of non-perturbative dynamics is neglected, the size of which is hard to estimate. Therefore, people may question the validity of the "15% rule" as a serious benchmark for comparing experimental data.

We present here two approaches to estimate the ratio Q_h, using the data as displayed in Table I-III. The first approach is based on an assumption that the decays of the J/ψ and $\psi(2S)$ in the lowest order of QCD are classified into hadronic decays (ggg), electromagnetic decays (γ^*), radiative decays into light hadrons (γgg), and decays to lower mass charmonium states($c\overline{c}X$) [9,10]. Thus, using the relation $B(ggg) + B(\gamma gg) + B(\gamma^*) + B(c\overline{c}X) = 1$, one can derive B(ggg)+B(γgg) by subtracting B(γ^*) and B($c\overline{c}X$) from unity.

The electromagnetic decay channels of the J/ψ produce hadrons, e^+e^- and $\mu^+\mu^-$ as final states. Besides these channels, the electromagnetic decays of $\psi(2S)$ also include the $\tau^+\tau^-$ as a final state. The experimental data is summarized in Table 1, where the branching fraction of $\psi(2S) \to \tau^+\tau^-$ is a recent measurement of the BES experiment [11], whereas the other data was taken from the Particle Data Group [4,12]. The total contributions to the electromagnetic decays of J/ψ and $\psi(2S)$ are then given as $B(J/\psi \to \gamma^*) = (28.81 \pm 2.00)\%$ and $B(\psi(2S) \to \gamma^*) = (5.08 \pm 0.55)\%$, respectively.

As regards to the decay into lower mass charmonium states, the J/ψ has only one radiative decay channel into η_c, whereas the $\psi(2S)$ can decay into a number of other final states, i.e. $\pi^+\pi^-J/\psi$, $\pi^0\pi^0J/\psi$, π^0J/ψ, $\eta J/\psi$, $\gamma\chi_{c0}$, $\gamma\chi_{c1}$, $\gamma\chi_{c2}$, $\gamma\eta_c$, $\gamma\eta_c(2S)$ and $1^1P_1 + X$. The decay rates of the last two channels are faint and thus are neglected in our calculation. The experimental data summarized in Table 2 are all taken from PDG [4]. Using this data we calculated the total contribution to $c\overline{c}X$: $B(J/\psi \to c\overline{c}X) = (1.3 \pm 0.4)\%$ and $B(\psi(2S) \to c\overline{c}X) = (78.1 \pm 3.9)\%$, respectively.

By deducting the contributions B(γ^*) and B($c\overline{c}X$), we find that $B(J/\psi \to ggg) + B(J/\psi \to \gamma gg) = (69.9 \pm 2.0)\%$ and $B(\psi(2S) \to ggg) + B(\psi(2S) \to \gamma gg) = (16.8 \pm 3.9)\%$. Therefore the ratio of branching fractions of $\psi(2S)$ to J/ψ decays into hadrons is given by

$$Q_1 = \frac{B(\psi(2S) \to ggg) + B(\psi(2S) \to \gamma gg)}{B(J/\psi \to ggg) + B(J/\psi \to \gamma gg)} = (24.0 \pm 5.6)\%. \tag{3}$$

The relation between the decay rates of ggg and γgg is readily calculated in PQCD to the first order as [7]

$$\frac{\Gamma(J/\psi \to \gamma gg)}{\Gamma(J/\psi \to ggg)} = \frac{16}{5}\frac{\alpha}{\alpha_s(m_c)}(1 - 2.9\frac{\alpha_s}{\pi}). \tag{4}$$

Using $\alpha_s(m_c) = 0.28$, one can estimate $\Gamma(J/\psi \to \gamma gg)/\Gamma(J/\psi \to ggg) \simeq 0.062$. A similar relation can be deduced for the $\psi(2S)$ decays. Thus one expects that "24% ratio" stands well for either ggg mode or γgg mode.

The other approach is to use the data on branching fractions for hadronic decays in final states containing pions, kaons, and protons that have already been measured for both J/ψ and $\psi(2S)$. They are $3(\pi^+\pi^-)\pi^0$, $2(\pi^+\pi^-)\pi^0$, $\pi^+\pi^-\pi^0$, $\pi^+\pi^-K^+K^-$, $\pi^+\pi^-P\overline{P}$, $P\overline{P}$, $P\overline{P}\pi^0$ and K^+K^-. Using the PDG data compiled in Table 3, we have $\sum_{i=1}^{8} B_i(J/\psi \to f_i) = (9.43 \pm 0.72)\%$ and $\sum_{i=1}^{8} B_i(\psi(2S) \to f_i) = (0.941 \pm 0.185)\%$. It follows that

$$Q_2 = \sum_{i=1}^{8} B_i(\psi(2S) \to f_i)/\sum_{i=1}^{8} B_i(J/\psi \to f_i) = (10.0 \pm 2.1)\%. \tag{5}$$

We note that the results obtained by the two approaches vary considerably. However, a comparison of the values of the total branching fraction for $\psi(2S) \to ggg$ computed by the two approaches indicates that only a small fraction ($\sim 6\%$) of the exclusive hadronic decays of $\psi(2S)$ have been reconstructed experimentally. It is thus obvious that Q_2 is not the exact ratio of $\psi(2S)$ to J/ψ inclusive hadronic decay rates, but represents on average the ratio of the exclusive decay channels, as measured to date. We therefore do not consider Q_2 any further. Nevertheless, the question persists as to where the remaining hadronic $\psi(2S)$ decay modes are and how the corresponding pattern of decays for $\psi(2S)$ to J/ψ behaves. It would be an intriguing experiment task to search for those remaining channels that are in such final states as those with higher multiplicities, or those with multi-neutral particles, or even for remaining channels in non-$q\overline{q}$ states. In comparison with the naive PQCD expectations, the central value of Q_1 is about a factor of two higher than that of Q_{th}, as stated in Eq. (2). However, the difference lies within the 2σ error of Q_1 and is

only marginally significant. The substantial error of Q_1 is essentially due to the propagation of errors during the subtraction of the decay rate $B(\psi(2S) \to c\overline{c}X)$ from the total rate, although the total error of $B(\psi(2S) \to c\overline{c}X)$ itself merely amounts to about 5%. Taking into account the apparent approximations to the naive expectations of PQCD as well, it seems to us premature to regard this as a remarkable discrepancy that deserves serious considerations.

To use the ratio Q_1 for comparison with the experimental results of $\psi(2S)$ and J/ψ decays, it should be noted that the estimate of Q_1, like the prediction of Q_{th}, is made for the total width for ggg decay, not for the partial widths of exclusive final states. Consequently, a number of corrections may be associated with specific exclusive decay modes: (I) It is shown that the J/ψ and $\psi(2S)$ decays to ωf_2, ρa_2, $K^{*0}\overline{K}_2^{*0} + c.c.$ and $\phi f_2'(1525)$ are hadron helicity conservation (HHC) allowed [13], while that to $\rho\pi$ and $K^*\overline{K}$ are HHC forbidden [14]. The general validity of the HHC at the charmonium mass scale is still an open question. It is suggested that a critical test would be to measure the angular distributions of exclusive final states [14]. Existing measurements on angular distributions for J/ψ decays into $p\overline{p}$, $\Lambda\overline{\Lambda}$, $\Sigma^0\overline{\Sigma}^0$ and $K_S^0K_L^0$ [15] are consistent with the HHC predictions (baryon pairs within $1-1.5$ standard deviations); however, no data is available for $\psi(2S)$ decays so far. Exclusive processes that violate helicity conservation are suppressed by powers of m^2/s in QCD [14]. This would contribute a suppression factor $M^2_{J/\psi}/M^2_{\psi(2S)} = 0.71$ to the ratio of the $\psi(2S)$ to J/ψ decay rates for final states $\rho\pi$ and $K^*\overline{K}$. On the other hand, the $\gamma\eta$ and $\gamma\eta'$ modes are allowed by the helicity selection rule, since helicity conservation applies only to the hadrons [13]. (II) Exclusive reactions which involve hadrons with quarks or gluons in higher orbital angular momentum states are suppressed by powers of $1/s = E_{cm}^{-2}$ [14]. This contributes a suppression factor of $M^2_{J/\psi}/M^2_{\psi(2S)}$ to the ratio of $\psi(2S)$ vs. J/ψ branching fractions for decays into such final states as ωf_2, ρa_2, $K^{*0}\overline{K}_2^{*0} + c.c.$ and $\phi f_2'(1525)$, where a meson in P-wave state is included. (III) Ref. [14] reported another suppression arising from the asymptotic form factor which would be $M^8_{J/\psi}/M^8_{\psi(2S)} = 0.25$ for decays to $p\overline{p}$ channel. Contrary to these calculations, Ref. [16] evaluated the three-gluon contribution with the c-quark mass instead of the charmonium mass. As a consequence, the ratio of the J/ψ and $\psi(2S)$ decay widths is not scaled to the 8th power of the ratio of their masses in our calculations.

Table 4 lists corrections to the ratio Q_1 for several exclusive hadronic decay channels. Experimental data from PDG [4] are also included for comparison. As seen from the table, the predicted corrected value of the ratio Q_1 for $b_1\pi$ is consistent with the experimental data, whereas the experimental ratio of $K_1^{\pm}(1270)K^{\mp}$ is enhanced as compared with the predicted value. The deviations of the measured ratios for decays into VP, VT and other final states (even $p\overline{p}$) from the corrected ratios demonstrate suppressions, in this case. Note that the combination of all the above correction results lead to a substantial reduction of the ratio of $\psi(2S)$ to J/ψ decay rates, well below 24% for many of the exclusive hadronic decay channels (this is also compatible with the observation mentioned above that the value of Q_2 is much lower than that of Q_1). One should therefore conclude that a considerable number of other decay channels ought to have an enhancement with a ratio above 24%, in order to make up for all these suppressed channels. It is puzzling that so far there has only been one channel for the $\psi(2S)$ decays observed, the $K_1^{\pm}(1270)K^{\mp}$ channel, which is enhanced relative to the J/ψ [18]. Further systematic study of $\psi(2S)$ decays are anxiously awaited.

As is seen from the above analysis, we have restricted our comparison only to decays to two-body final states. What if one makes a comparison for decays leading to three or more hadrons? MARKII did make such a comparison in their original work [2] and claimed that the ratio Q for decay modes such as $p\overline{p}\pi^0$, $p\overline{p}\pi^+\pi^-$, $K^+K^-\pi^+\pi^-$, $2(\pi^+\pi^-)\pi^0$, and $3(\pi^+\pi^-)\pi^0$, is consistent with the naive theoretical expectations. However, it should be pointed out that most of these multi-hadron final states in fact include sums of several two-body intermediate states. One thus observes a mixed effect which may not deviate noticeably from the expected value of Q, even if a few of the two-body intermediate states are severely suppressed. For example, the decay $J/\psi \to 2(\pi^+\pi^-)\pi^0$ proceeds predominantly through intermediate states $b_1\pi$, ωf_2, and $a_2\rho$. The observed Q for this decay, as reported by MARKII [2], was (9.5 ± 2.7)%, does deviate, though not quite significantly, from the "15% rule". This results from the fact that two of these three two-body intermediate states are found to be anomalously suppressed [17]. Therefore, one must be always cautious about drawing conclusions from the comparison of decays of multi-hadron final states.

An experimental situation similar to the $\rho\pi$ puzzle occurs in the decays of the η_c in three vector meson (VV) cases, such as $\rho\rho$, $K^*\overline{K}^*$ and $\phi\phi$, and in $p\overline{p}$. These decays are all first-order forbidden by HHC in PQCD [14,20]; however, they are actually observed to occur with relatively large branching fractions [4]. It is thus interesting to look for the analogous decays of the $\eta_c(2S)$ and compare the ratio of the $\eta_c(2S)$ to η_c branching fractions with the relation $B(\eta_c(2S) \to h) \simeq B(\eta_c \to h)$ predicted by Chao et al [21]. In testing for helicity conservation, these decay modes for η_c and its spin-singlet partner $\eta_c(2S)$ play the same role as the decay modes $\rho\pi$ and $K^*\overline{K}$ do in the case of J/ψ and its spin-triplet partner $\psi(2S)$. The search for the $\eta_c(2S)$ is thus important not only because the $\eta_c(2S)$ is one of the two remaining states of the charmonium family awaiting confirmation (or discovery) but also because the study of its hadronic decays could shed light on the puzzle of J/ψ and $\psi(2S)$ decays.

We now move on to discuss various theoretical models made to explain the $\rho\pi$ puzzle as it is presently formulated. Instead of a critical examination of many theoretical arguments (which one can find in the literature [1,22–24]), we will concentrate exclusively on comparing the experimental results, mostly from the BES experiment, to the predictions of these models, in an attempt to differentiate between them.

The first explanation for the MARKII observation, as proposed by Hou and Soni [25] and generalized later by Brodsky et al. [26], is the postulate that the decay J/ψ, which violates the helicity selection rule of PQCD, is enhanced by the mixing of the J/ψ with a vector glueball O that decays preferentially to $\rho\pi$ and other VP channels. The vector glueball O is required to be fairly narrow and nearly degenerate with the J/ψ. The BES has searched for this hypothetical particle in a $\rho\pi$ scan across the J/ψ region in e^+e^- annihilations as well as in decays $\psi(2S) \to \pi\pi O$, $O \to \rho\pi$, and found no evidence for its existence [27,28]. The data constrains the mass and width of the O to the range $\mid m_O - m_{J/\psi} \mid < 80MeV$ and $4MeV < \Gamma_O < 50MeV$ [3]. This mass, as indicated in Ref. [29], is several hundred MeV lower than the lightest vector glueball observed in lattice simulations of QCD without dynamical quarks. More recently, a few more experimental facts unfavorable to this model have been reported by BES. One is the identification of isospin-violating VP mode $\psi(2S) \to \omega\pi^0$ with a large branching fraction [3]. This contradicts the essence of the model that the pattern of suppression is dependent on the spin-parity of the final state mesons. The other is the finding of suppression of $\psi(2S)$ decays into vector plus tensor (VT) final states [17]. Since hadronic VT decays, unlike the VP decays, conserve HHC, some other mechanism must be responsible for this suppression in the model. Furthermore, it has been argued that the O may also explain why J/ψ decays to ϕf_0 (named previously S^*) but not to $\rho\delta$, since the O mixes with the ϕ and enhances a mode that would be otherwise suppressed [26]. However, the observation of non-suppressed $\psi(2S) \to \phi f_0$ [3], which implies the absence of anomalous enhancement in $J/\psi \to \phi f_0$, would rule out such an explanation. Anselmino et al. extended the idea of $J/\psi - O$ mixing to the case of $\eta_c \to$VV and $p\overline{p}$ [20]. They suggested that the enhancement of these decays can be attributed to the presence of a trigluonium pseudoscalar state with a mass not far from the η_c mass. So far no experimental data has supported the existence of such a state.

Recently Brodsky and Karliner proposed the existence of intrinsic charm $\mid \overline{q}q\overline{c}c >$ Fock components of the light vector mesons as another mechanism to account for the J/ψ decays to VP channels and their suppression of $\psi(2S)$ [30]. They also suggested comparing branching fractions for the η_c and $\eta_c(2S)$ as clues to the importance of η_c intrinsic charm excitations in the wavefunctions of light hadrons. However, the BES observation of $\psi(2S) \to \omega\pi^0$ would again appear to disfavor this model.

Chaichian and Törnqvist suggested a model which invokes a form factor falling exponentially with the energy to suppress all $\psi(2S)$ decays to lowest-lying two-body meson final states [31]. However, the BES report on observation of a number of $\psi(2S)$ hadronic two-body decays such as $b_1\pi$, ϕf_0, $K_1(1270)\overline{K}$, and $\omega\pi^0$ have proved the contrary [18,3]. In addition, the BES upper limit at 90% C.L. $B(\psi(2S) \to \rho\pi) < 2.8 \times 10^{-5}$ [3] is well below the branching fraction predicted by this model, 7×10^{-5}.

The generalized hindered M1 transition mechanism proposed by Pinsky [32] relates the process $\psi(2S) \to \gamma\eta'$ to the hindered M1 transition $\psi(2S) \to \gamma\eta_c$. This predicts $Q_{\gamma\eta'}$ to be 2×10^{-3}, which, as already shown in Ref. [19], falls more than an order of magnitude below the BES data $(3.6 \pm 0.9) \times 10^{-2}$. According to this model, the hadronic decays of $\psi(2S)$ to VP final states are also a generalized hindered M1 transition. The branching fraction for the decay of $\psi(2S) \to \rho\pi$ is estimated to be 4×10^{-5}, as compared to the measured limit of 2.8×10^{-5}. Moreover, it is inferred from this model that $\psi(2S) \to \gamma f_2$ decay should be suppressed whereas $\psi(2S) \to \omega f_2$ should not [22]. However, the experimental facts from BES contradict this assumption [17,3].

Karl and Roberts have suggested a proposal to explain the $\rho\pi$ puzzle based on the mechanism of sequential quark pair creation [33]. Even though their predictions could generally accommodate the data for decays of J/ψ and $\psi(2S)$ to $\rho\pi$ or to $K^*\overline{K}$, it seems hard to explain the large branching fraction for ϕ decays to $\rho\pi$ [4] due to the fact that their fragmentation probability tends to zero as the mass of the $\rho\pi$ decaying system approaches 1GeV.

More recently, Li et al. [34] pointed out that final-state interactions in J/ψ and $\psi(2S)$ decays give rise to effects which are of the same order as the tree level amplitudes, and may be a possible explanation for all the observed suppressed modes of $\psi(2S)$ decays including $\rho\pi$, $K^*\overline{K}$ and ωf_2. They thus predicted qualitatively large production rates of $a_1\rho$ and $K_1^*\overline{K}^*$ for $\psi(2S)$, the verification of which may give further support to their model. So far, BES has never reported such measurements; nevertheless, useful information on $a_1\rho$ and $K_1^*\overline{K}^*$ could be obtained from its published data as shown in Ref. [17]. The lack of evidence within the invariant mass distribution plots (see Fig.3 and Fig.5 of Ref. [17]) that the $\rho\pi$ recoiled against a ρ for events of $\psi(2S) \to \rho^0\rho^\pm\pi^\mp$ and that $\pi^\pm K^\mp$ recoiled against a K^{*0} for events of $\psi(2S) \to \pi^+\pi^-K^+K^-$ suggests that they are unlikely to be the favored modes in $\psi(2S)$ decays.

A model put forward by Gérald and Weyers entertains the assumption that the three-gluon annihilation amplitude and the QED amplitude add incoherently in all channels in J/ψ decays into light hadrons, while in the case of $\psi(2S)$

decays the dominant QCD annihilation amplitude is not into three gluons, but, via a two step process, into a specific configuration of five gluons [35]. Besides explaining the measurements on $\psi(2S)$ decays to $\rho\pi$, $K^*\overline{K}$, and $\omega\pi^0$, this model predicts a sizeable $\psi(2S) \to (\pi^+\pi^-$ or $\eta)h_1(1170)$ branching fraction. Indeed the BES has performed extensive analysis of decays $\psi(2S) \to \pi\pi\rho\pi$ to look for new particles; however, it is unlikely that a conclusive signal for $h_1(1170)$ has ever been observed in the inclusive spectrum of $\psi(2S)$ decays to $\pi\pi\rho\pi$ [28].

Chen and Braaten proposed an explanation [29] for the $\rho\pi$ puzzle, arguing that the decay $J/\psi \to \rho\pi$ is dominated by a Fock state in which the $c\overline{c}$ pair is in a color-octet $^3S_1$ state which decays via $c\overline{c} \to q\overline{q}$, while the suppression of this decay mode for the $\psi(2S)$ is attributed to a dynamical effect due to the small energy gap between the mass of the $\psi(2S)$ and the $D\overline{D}$ threshold. Using the BES data on the branching fractions into $\rho\pi$ and $K^*\overline{K}$ as input, they predicted the branching fractions for many other VP decay modes of the $\psi(2S)$. Most recently, Feldmann and Kroll parametrized the strong interaction mechanism for the hadron-helicity non-conserving decays in a similar way, but interpreted it differently [23]. They argued that, for these processes, the charmonium state decays through a light-quark Fock component by a soft mechanism, which is characteristic of OZI-rule allowed strong decays. Estimating the light-quark admixture by meson-mixing, they also obtained a numerical description of the branching fractions for many VP decay modes of the J/ψ and $\psi(2S)$. The predictions of both models are in good agreement with the measured branching fractions (some are preliminary) from the BES experiment [3] as well as the PDG data [4]. Chen and Braaten's proposal also has implications for the angular distributions for two-body decay modes of $\psi(2S)$. Nevertheless, such measurements would be extremely difficult, if not impossible, to perform for those strongly suppressed decay modes in $\psi(2S)$ decays. Feldmann and Kroll, on the other hand, has extended their mixing approach to the $\eta_c \to$ VV decays and obtained a reasonable description of the branching fractions for these decays while the $\eta_c(2S) \to$ VV decays are expected to be strongly suppressed.

From the above discussion we see that essentially none of the models are able to explain all known experimental results; in particular no analysis on the suppression of $\psi(2S) \to$ VT decays has been given. Not a few models appear to have more assumptions than predictions, not to mention quantitative predictions. While the current data seem to rule out convincingly some of the models, a few other models may warrant further consideration; for them both detailed theoretical analyses and additional experimental tests are demanded. It seems to us that a key premise for physical considerations is to establish whether the J/ψ decays or the $\psi(2S)$ decays are anomalous. An amplitude analysis made for the two-body decays of J/ψ to VP [24] has shown that nothing anomalous is found in the magnitudes of the three-gluon and one-photon decay amplitudes. If this is sustained, those arguments presupposing the J/ψ as the origin of the anomaly should be disregarded.

In summary, we have examined the $\rho\pi$ puzzle of J/ψ and $\psi(2S)$ decays in the light of current experimental data. The estimates of the ratio of $\psi(2S)$ to J/ψ hadronic decay rates, using two different approaches, differ substantially from each other. The one using only the data of exclusive hadronic decays appears to be underestimated. The other estimate, that is based on computation of the inclusive ggg decay rate by subtracting other decay rates from the total decay rate, differs by 2σ-error of this estimate from the naive prediction of PQCD, even though its central value is about a factor of two as large as the latter. By comparing this estimated ratio, and taking account the corrections associated with specific exclusive decay modes with the corresponding experimental data, anomalies in J/ψ and $\psi(2S)$ decays to VP, VT, AP and some other final states are evident. We found from our analysis that the exclusive hadronic decays of the $\psi(2S)$ so far reconstructed experimentally account for only a small fraction of the total $\psi(2S)$ decays and a ratio of $\psi(2S)$ to J/ψ hadronic decay rates that exceeds the estimated value is expect to occur for a considerable part of the remaining $\psi(2S)$ decay channels. We have also shown that the recent new results from the BES experiment provide crucial tests of various theoretical models proposed to understand the $\rho\pi$ puzzle. Further experimental and theoretical efforts are required, in order to fill the missing data and definitively solve the perplexing $\rho\pi$ puzzle.

The authors wish to thank M. Wise, F. Porter, S. J. Brodsky, G. D. Zhao, S. F. Tuan and J. Weyers for enlightening discussions. The discussions with the BES collaboration are appreciatively acknowledged. This work was supported by the National Natural Science Foundation of China under Contract No. 19290400, the Chinese Academy of Sciences under Contracts No. H-10 and No. E-01, and by the U.S. Department of Energy under Contract No. DE-FG03-86ER40271.

† Email: guyf@hpws3.ihep.ac.cn.

[1] See for example, S. F. Tuan, Commun. Theor. Phys. **33**, 285 (2000); Y. F. Gu and S. F. Tuan, Nucl. Phys. A **675**, 1-2, 404 (2000).
[2] M. E. B. Franklin et al., Phys. Rev. Lett. **51**, 963 (1983).
[3] F. A. Harris, hep-ex/9903036 (unpublished); F. Liu, hep-ex/9912006 (unpublished).
[4] D. E. Groom *et al.* (Particle Data Group), Eur. Phys. Jour. C15, 1 (2000).
[5] T. Applequist and D. Politzer, Phys. Rev. Lett. **34**, 43 (1975).
[6] The factor $\alpha_s^3(\psi(2S))/\alpha_s^3(J/\psi) = 0.845$ is calculated by using Eq. (9.5a) in [4], where $n_f = 4$ for charmonium energy region and the QCD scale factor $\Lambda^{(4)}_{\overline{MS}} = (209 \pm 21)\ MeV$ from J. Z. Bai et al., Phys. Rev. D **58**, 092006 (1998) are adopted.
[7] W. Kwong, Phys. Rev. D **37**, 3210 (1988).
[8] H. C. Chiang et al., Phys. Lett. B **324**, 482 (1994).
[9] L. Köpke and N. Wermes, Phys. Reports **174**, 67 (1989).
[10] An earlier estimate of Q by the same approach is given in Y. F. Gu, in Proc. Intern. Workshop on Hadron Physics at e^+e^- Colliders, Beijing, China, ed. X. L. Fan et al. (IHEP, Beijing, 1997) p. 69.
[11] J. Z. Bai et al., hep-ex/0010072 (unpublished).
[12] More accurate values for the branching fractions of $J/\psi, \psi(2S) \to \gamma^* \to hadrons$ can be obtained by making the perturbative correction of $(1 + \overline{\alpha}_s/\pi)$ to the leptonic branching fractions. However, we do not use them to replace the old measurements quoted in PDG [4] due to the poor knowledge of the theoretical uncertainties.
[13] Y. F. Gu and S. F. Tuan, Mod. Phys. Lett. A **10**, 615 (1995).
[14] S. J. Brodsky and G. P. Lepage, Phys. Rev. D **24**, 2848 (1981); Perturbative Quantum Chromodynamics, World Scientific, Singapore, 1989.
[15] I. Peruzzi et al., Phys. Rev. D **17**, 2901 (1978); R. Brandelik et al., Z. Phys. C **1**, 233 (1979); M. W. Eaton et al., Phys. Rev. D **29**, 804 (1984); D. Pallin et al., Nucl. Phys. B **292**, 653 (1987); J. S. Brown, Ph.D. thesis, Univ. of Washington, UMI 84-19117-MC (unpublished); R. M. Baltrusaitis et al., Phys. Rev. D **32**, 566 (1985).
[16] J. Bolz and P. Kroll, Eur. Phys. J. C **2**, 545 (1998); P. Kroll, Nucl. Phys. Proc. Suppl. **64**, 456 (1998); P. Kroll, Acta Phys. Polon. B **29** 2789 (1998).
[17] J. Z. Bai et al., Phys. Rev. Lett. **81**, 5080 (1998).
[18] J. Z. Bai et al., Phys. Rev. Lett. **83**, 1918 (1999).
[19] J. Z. Bai et al., Phys. Rev. D **58**, 097101 (1998).
[20] M. Anselmino, M. Genovese, and E. Predazzi, Phys. Rev. D **44**, 1597 (1991); M. Anselmino, M. Genovese, and D. E. Kharzeev, ibid. **50**, 595 (1994).
[21] Kuang-Ta Chao, Y. F. Gu, and S. F. Tuan, Commun. Theor. Phys. **25**, 471 (1996)
[22] K.-T. Chao, Proc. 17th Intern. Symp. on Lepton-Photon Interactions at high Energies, Beijing, China, 1995, ed. Z. P. Zheng and H. S. Chen (World Scientific, Singapore, 1996) p. 106.
[23] T. Feldmann and P. Kroll, Phys. Rev. **D** 62 (2000) 074006.
[24] M. Suzuki, Phys. Rev. D **57**, 5717 (1998).
[25] Wei-Shu Hou and A. Soni, Phys. Rev. Lett. **50**, 569 (1983).
[26] S. J. Brodsky et al., Phys. Rev. Lett. **59**, 631 (1987).
[27] J. Z. Bai et al., Phys. Rev. D **54**, 1221 (1996); Erratum: ibid. **57**, 3187 (1998).
[28] Y. H. Xie, Ph.D. thesis, IHEP, Chinese Academy of Sciences, 1998 (unpublished).
[29] Yu-Qi Chen and Eric Braaten, Phys. Rev. Lett. **80**, 5060 (1998).
[30] S. J. Brodsky and M. Karliner, Phys. Rev. Lett. **78**, 468 (1997).
[31] M. Chaichian and Törnqvist, Nucl. Phys. B **323**, 75 (1989).
[32] S. S. Pinsky, Phys. Lett. B **236**, 479 (1990).
[33] G. Karl and W. Roberts, Phys. Lett. B **144**, 243 (1984).
[34] X.-Q. Li, D. V. Bugg, and B.-S. Zou, Phys. Rev. D **55**, 1241 (1997).
[35] J.-M. Gérard and J. Weyers, Phys. Lett. B **462**, 324 (1999).

Table 1. Experimental data on branching fractions for electromagnetic decays of J/ψ and $\psi(2S)$ used in our analysis. All data are taken from PDG [4] except the branching fraction for $\psi(2S) \to \tau^+\tau^-$ which is a first measurement by BES [11].

Channel	B(J/ψ)	B($\psi(2S)$)
$\gamma^* \to hadrons$	$(17.0 \pm 2.0)\%$	$(2.9 \pm 0.4)\%$
e^+e^-	$(5.93 \pm 0.10)\%$	$(8.8 \pm 1.3) \times 10^{-3}$
$\mu^+\mu^-$	$(5.88 \pm 0.10)\%$	$(1.03 \pm 0.35)\%$
$\tau^+\tau^-$		$(2.71 \pm 0.70) \times 10^{-3}$

Table 2. Experimental data on branching fractions for J/ψ and $\psi(2S)$ decays to lower mass charmonium states used in our analysis. All data are taken from PDG [4].

Channel	B(J/ψ)	B($\psi(2S)$)
$\gamma\eta_c$	$(1.3 \pm 0.4)\%$	$(0.28 \pm 0.06)\%$
$\pi^+\pi^- J/\psi$		$(31.0 \pm 2.8)\%$
$\pi^0\pi^0 J/\psi$		$(18.2 \pm 2.3)\%$
$\eta J/\psi$		$(2.7 \pm 0.4)\%$
$\pi^0 J/\psi$		$(9.7 \pm 2.1) \times 10^{-4}$
$\gamma\chi_{c0}$		$(9.3 \pm 0.9)\%$
$\gamma\chi_{c1}$		$(8.7 \pm 0.8)\%$
$\gamma\chi_{c2}$		$(7.8 \pm 0.8)\%$

Table 3. Branching fractions for the J/ψ and $\psi(2S)$ exclusive hadronic decays used in our analysis. All data are from PDG [4].

Mode	B(J/ψ)	B($\psi(2S)$)
$\pi^+\pi^-\pi^0$	$(1.50 \pm 0.20)\%$	$(8 \pm 5) \times 10^{-5}$
$2(\pi^+\pi^-)\pi^0$	$(3.37 \pm 0.26)\%$	$(3.0 \pm 0.8) \times 10^{-3}$
$3(\pi^+\pi^-)\pi^0$	$(2.9 \pm 0.6)\%$	$(3.5 \pm 1.6) \times 10^{-3}$
$K^+K^-\pi^+\pi^-$	$(7.2 \pm 2.3) \times 10^{-3}$	$(1.6 \pm 0.4) \times 10^{-3}$
$p\overline{p}\pi^+\pi^-$	$(6.0 \pm 0.5) \times 10^{-3}$	$(8.0 \pm 2.0) \times 10^{-4}$
$p\overline{p}\pi^0$	$(1.09 \pm 0.09) \times 10^{-3}$	$(1.4 \pm 0.5) \times 10^{-4}$
$p\overline{p}$	$(2.12 \pm 0.10) \times 10^{-3}$	$(1.9 \pm 0.5) \times 10^{-4}$
K^+K^-	$(2.37 \pm 0.31) \times 10^{-4}$	$(1.0 \pm 0.7) \times 10^{-4}$

Table 4. The ratio of branching fractions of $\psi(2S)$ and J/ψ exclusive decays: $Q_f = B(\psi(2S))/B(J/\psi)$. All data are from PDG [4]. Upper limits are given at the 90% confidence level.

mode	HHC	orbital momentum	pred. Q_f (%)	meas. Q_f (%)
$P\overline{P}$	1	1	24.0 ± 5.6	9.0 ± 2.4
$\rho\pi$	0.71	1	17.0 ± 4.0	< 0.65
$K^+\overline{K}^*(892)^-$	0.71	1	17.0 ± 4.0	< 1.1
$\omega f_2(1270)$	1	0.71	17.0 ± 4.0	< 4.0
$\rho a_2(1320)$	1	0.71	17.0 ± 4.0	< 2.1
$K^*(892)^0\overline{K}_2^*(1430)^0$	1	0.71	17.0 ± 4.0	< 1.8
$\phi f_2'(1525)$	1	0.71	17.0 ± 4.0	< 5.6
$b_1^\pm\pi^\mp$	1	0.71	17.0 ± 4.0	17.3 ± 5.2
$K_1^\pm(1270)K^\mp$	1	0.71	17.0 ± 4.0	> 33.3
$K_1^\pm(1400)K^\mp$	1	0.71	17.0 ± 4.0	< 8.2
$\gamma\eta'(958)$	1	1	24.0 ± 5.6	3.5 ± 1.0
$\gamma\eta$	1	1	24.0 ± 5.6	< 10.5